Physics of Non-Inertial Reference Frames

Timur F. Kamalov
Physics Department, Moscow State Open University
Korchagina, 22, Moscow, 107996, Russia
E-mail: TimKamalov@mail.ru, TimKamalov@gmail.com

Physics of non-inertial reference frames is a generalizing of Newton's laws to any reference frames. The first, Law of Kinematic in non-inertial reference frames reads: the kinematic state of a body free of forces conserves and determinates a constant n-th order derivative with respect to time being equal in absolute value to an invariant of the observer's reference frame. The second, Law of Dynamic extended Newton's second law to non-inertial reference frames and also contains additional variables there are higher derivatives of coordinates. Dynamics Law in non-inertial reference frames reads: a force induces a change in the kinematic state of the body and is proportional to the rate of its change. It is mean that if the kinematic invariant of the reference frame is n-th derivative with respect the time, then the dynamics of a body being affected by the force F is described by the (n+1)-th differential equation. The third, Law of Static in non-inertial reference frames reads: the sum of all forces acting a body at rest is equal to zero.

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Newton's laws are valid in inertial reference frames, with the Lagrangian being dependent on coordinates and their first derivatives (i.e. velocities). A mathematician would call three Newton's laws an axiomatic of physical theory. However there exists Ostrogradski's Canonical Formalism. This mathematical description, in which we shall try to find a physical meaning, will be called Ostrogradski's physics. For this we formulate two postulates. The Lagrangian depends not only on coordinates and their first derivatives (i.e. velocities) but also on higher derivatives of coordinates, with he higher derivatives being considered independent variables. Really, if the first derivative of coordinates can be independent of coordinates, why cannot the higher derivatives? Repeating the well known procedure of obtaining Euler-Lagrange's equation for such a Lagrangian, we shall obtain Euler-Lagrange's equation with additional variables depending on the higher derivatives of coordinates. Ostrogradski's physics is a physics of non-inertial reference frames. This is the theory with a different axiomatic, and hence it gives different results. What axiomatic of non-inertial reference frames physics should be? The first axiom reads: free particles conserves its kinematical states in any reference frame including an inertial one. The second axiom generalized Newton's second law to any reference frames and also contains additional variables there are higher derivatives of coordinates and inertial forces, since Ostrogradski's physics considers any reference frames including an inertial one. Newton's physics is a particular case of Ostrogradski's Canonical Formalism for inertial reference frames.

Classical physics usually considers the motion of bodies in inertial reference frames. This is a simplified and approximate description of the real pattern

of the motion, as it is practically impossible to get an ideal inertial reference frame. Actually in the any particular reference frame there always exist minor influences due to any random fields. A simplified consideration of the actual reference frame as an inertial one enables derivation of motion equations, which are usually solved by means of the traditional methods of mathematical physics. Then the uncertainty principle is induced by the non-inertial character of the reference frame and constitutes the expression of the error of measurement of coordinate and momentum of the object under consideration and is a consequence of the idealization of the problem being considered to an inertial reference frame. In this case, one can assess the effect of inertial force in a non-inertial reference frame through the Plank constant.

Let us consider the precise description of the dynamics of the motion of bodies taking into account complex non-inertial nature of reference frames. For this end, let us consider a body in a any reference frame, denoting the actual position of the body as r, actual momentum as p and time as t. Then, expanding into Taylor series the function $r = r(t)$ and $p = p(t)$, we get

$$r = r_0 + vt + \frac{at^2}{2} + \frac{1}{3!}\dot{a}t^3 + \frac{1}{4!}\ddot{a}t^4 + ... + \frac{1}{n!}\overset{.(n)}{r}t^n + ... \tag{1}$$

$$p = p_0 + \dot{p}t + \frac{\ddot{p}t^2}{2} + \frac{1}{3!}\dddot{p}t^3 + ... + \frac{1}{n!}\overset{.(n)}{p}t^n + ... \tag{2}$$

The kinematical state of the particle is define when any derivatives of coordinates on time equal to zero, i.e. $\overset{.(n)}{r} = 0$. Then the derivative of coordinates on time $\overset{.(n-1)}{r} = const$ is the kinematical invariant of the reference frame.

Let us compare this expansion with the well-known kinematical equation for inertial reference frames of Newtonian physics relating the distance to the acceleration a,

$$r_{inertial} = r_0 + vt + \frac{at^2}{2} \tag{3}$$

and the momentum

$$p_{inertial} = p_0. \tag{4}$$

Denoting the hidden (or correction and addition) variables accounting for additional terms in any reference frames with respect to inertial ones as Δr and Δp, we get

$$\Delta r = \frac{1}{3!}\dot{a}t^3 + \frac{1}{4!}\ddot{a}t^4 + ... + \frac{1}{n!}\overset{.(n)}{r}t^n + ... \tag{5}$$

$$\Delta p = \dot{p}t + \frac{\ddot{p}t^2}{2} + \frac{1}{3!}\dot{a}t^3 + \frac{1}{4!}\ddot{a}t^4 + ... + \frac{1}{n!}\overset{.(n)}{r}t^n + ... \tag{6}$$

Then

$$r_{any} = r_{inertial} + \Delta r \tag{7}$$

$$p_{any} = p_{inertial} + \Delta p. \tag{8}$$

In this case, the measurement error of an experiment follows from incompleteness of the description of sample particles in inertial reference frames, as we assume the actual space-and-time to be a non-inertial reference frame:

$$(p_{any} - p_{inertial})(r_{any} - r_{inertial}) \leq h, \tag{9}$$

$$\Delta p \Delta r \leq h, \tag{10}$$

h being the bound of correction's variables at the transformation of inertial to any reference frames. Then comparing this in equation with the uncertainty relation for $\Delta r = r- < r >$ and $\Delta p = p- < p >$ means uncertainty of coordinates and momentums of the particle in the process of measurements

$$\Delta p \Delta r \geq h, \tag{11}$$

we can expect that h is the some constant.

For non-inertial reference frames, the h constant accounts for the effect of the non-inertial space-and-time. Higher time derivatives of spatial coordinates act as hidden variables complementing the description of sample particles for inertial reference frames.

Newton's laws are valid in inertial reference frames with the Lagrangian L is the function of only the coordinates and their first derivatives (i.e. velocities), $L = L(t, r, \dot{r})$. For non-Newtonian physics in non-inertial reference frames, the Lagrangian depends on the coordinates and their higher derivatives as well as of the first one, i.e. $L = L(t, r, \dot{r}, \ddot{r}, \dddot{r}, ..., \overset{\cdot(n)}{r})$. Here the coordinates and their higher derivatives is independent. Applying the principle of least action, we get [3]

$$\delta S = \delta \int L(r, \dot{r}, \ddot{r}, \dddot{r}, ..., \overset{\cdot(n)}{r})dt = \int \sum_{n=0}^{N} (-1)^n \frac{d^n}{dt^n} (\frac{\partial L}{\partial \overset{\cdot(n)}{r}}) \delta r dt = 0. \tag{12}$$

Then, the Euler – Lagrange function for complex non-inertial reference frames takes on the form

$$\frac{\partial L}{dr} - \frac{d}{dt}(\frac{\partial L}{\partial \dot{r}}) + \frac{d^2}{dt^2}(\frac{\partial L}{\partial \ddot{r}}) - \frac{d^3}{dt^3}(\frac{\partial L}{\partial \dddot{r}}) + \frac{d^4}{dt^4}(\frac{\partial L}{\partial \overset{\cdot(4)}{r}}) + ... = 0 \tag{13}$$

It is the equation of the motion of particle with fee of forces influence in non-inertial reference frames

$$\sum_{n=0}^{N}(-1)^n \frac{d^n}{dt^n}(\frac{\partial L}{\partial \overset{\cdot(n)}{r}}) = 0 \tag{14}$$

Let us consider in more detail this precise description of the dynamics of body motion, taking into account of real reference systems. To describe the extended dynamics of a body in an arbitrary coordinate system (corresponding to any reference system) let us introduce concepts of kinematic state and kinematic invariant of an arbitrary reference system.

Definition: *the kinematic state of a body free of forces determinates a constant n-th order derivative with respect to time being equal in absolute value to an invariant of the observer's reference frame.*

Considering the dynamics of particles in any reference systems, we suggest the following important laws.

Law of Kinematic in non-inertial reference frames. *Kinematics Law in non-inertial reference frames reads: the kinematic state of a body free of forces conserves and determinates a constant n-th order derivative with respect to time being equal in absolute value to an invariant of the observer's reference frame. The kinematics state of a body is defined if the n-th derivative of its coordinate with respect to time is finite and equal to a negative value of the reference frame invariant.* That is,

$$q = q_0 + \dot{q}t + \frac{1}{2!}\ddot{q}t^2 + ... + \frac{1}{n!}\dot{q}^{(n)}t^n, \tag{15}$$

$$\frac{d^n q}{dt^n} = \dot{q}^{(n)} = const. \tag{16}$$

The acceleration for a body which free from forces influence is a constant for the observer in the uniformed reference frame. In this case the acceleration is defining the kinematic state of the body. The Law of Kinematics is the kind of equivalence principle and expresses the extension of the First Newton's Law to non-inertial reference frames. We can find so many planets in the Universe for this case. In the extended model of dynamics, the transition from a reference frame to another one is defined the transformation of reference frames as

$$q' = q_0 + \dot{q}t + \frac{1}{2!}\ddot{q}t^2 + ... + \frac{1}{n!}\dot{q}^{(n)}t^n \tag{17}$$

$$t' = t. \tag{18}$$

In this case Taylor's series decomposition of the coordinate must be convergence.

Law of Dynamic in non-inertial reference frames. *The force acting to the particle equal to the velocity of changing the kinematics state of the particle. If the kinematic invariant of an arbitrary reference frame is n-th time*

derivative of body coordinate, then the body dynamics with influence of the force $F(t, q, \dot{q}, \ddot{q}, ..., \dot{q}^{(n)})$ is described with the differential equation of the order $(n+1)$:

$$\alpha_{n+1}\dot{q}^{(n+1)} + ... + \alpha_2\ddot{q} + \alpha_1\dot{q} + \alpha_0 q = F(t, q, \dot{q}, \ddot{q}, ..., \dot{q}^{(n)}). \tag{19}$$

Here α_n- some constants.

Here (12) is the modification of the Newton's Second Law [1] for the general case of non-inertial reference frames. Odd derivatives correspond to losses (friction or radiation) and describe irreversible cases for open systems not satisfying variational principles of mechanics.

Law of Static in non-inertial reference frames. *In arbitrary reference frames the sum of forces which action to the statics particle is equal to zero.*

The Generalized Poisson's equation for the scalar potential φ of gravitational field in this case from the sources with density distribution of the source ρ and factor $\varkappa$ depending on the system of units shall take on the form

$$\sum_{n=0}^{N} {}^{n} \frac{\partial \varphi}{\partial \overset{\cdot(n)}{r}} = \varkappa\rho \tag{20}$$

or, in our case, Generalized Poisson's equation is

$$\textstyle\sum_{n=0}^{N} \overset{\cdot(n)}{\nabla} \varphi = \varkappa\rho.$$

Than the solution of Generalized Poisson's equation is

$$\varphi(t, r(t)) = \sum_{n=0}^{N} \varphi_0 \exp(-k/\overset{\cdot(n)}{r}(t)). \tag{21}$$

In the particular, for discussion the speculation of gravity the gravitational field the potential for example is

$$\varphi = \varphi_0 \exp(-k/r) = \frac{GM}{r} \exp(-k/r), \tag{22}$$

where φ- potential, G- gravitational constant, k - unknown constant or the scale of the interaction, $M = \int \rho dv$ - mass, $r = x - x_0 << 1$, x and x_0-coordinates. The constant k is unknown, but if k is equal to the Plank constant $l_p = 10^{-33}$ cm than this potential is always the same as Newtonian potential $\varphi = GM/r$. If constant k is equal to the size of nuclear $k = 10^{-15}m$ than the gravitational force is equal to nuclei forces because at the small distant gravitational forces is change on exponential law and be strong.

From this paper follow, that the phase space of coordinates and there multiple derivative gives the modification of the Newton's formula for the small scales for gravitational potential φ of two mass m is

$\varphi = \frac{Gm}{r}(1 - b\frac{k}{r} + c\frac{k^2}{r^2} - d\frac{k^3}{r^3} + ...)$,

where a,b,c,... - constants. In the particular case when $r >> k$ from (14) gravitational potential φ is

$$\varphi = \frac{GM}{r}(1 - \frac{k}{r} + \frac{k^2}{r^2} - \frac{k^3}{r^3} + ...) \quad (23)$$

Here k is the unknown constant which have the seance of distance. For example, if $k \sim 10^{-15}m$ and $r > k$ we have always Newtonian low.

For long distances $r >> k$, we have the equation for the Newtonian gravitational potential $\varphi_0 = Gm\frac{1}{r}$. For the distance $r < k$ the gravitational potential φ is a strong and in this case we can compare the gravitational force with the nuclear force. This modification of the Newton's gravitation law we can consider on the case of the dark matter.

For particle described by the generalized Hamilton function at small distances, i.e. when the series diverges, there shall be much stronger forces acting than it is usually considered in calculations employing the Hamilton function. This theory of short-range interaction explains interaction of bodies at small distances and refines the description of their interaction in case of their increase. It can be supposed that this method can be applied to cases when the force of gravitational attraction of particles described by the Hamilton function, at low distances.

Denoting the addition energy brought about by the non-inertial reference frame as Q and the constant coefficients as α_i, we get for the total energy E, potential energy V and kinetic energy W the following expressions:

$$E = \alpha_0 r^2 + \alpha_1 \dot{r}^2 + \alpha_2 \ddot{r}^2 + \alpha_3 \dddot{r}^2 + ... + \alpha_n \overset{\cdot(n)}{r}^2 + ...$$
$$E = V + W + Q$$
$$V = \alpha_0 r^2$$
$$W = \alpha_1 \dot{r}^2$$
$$Q = \alpha_2 \ddot{r}^2 + \alpha_3 \dddot{r}^2 + ... + \alpha_n \overset{\cdot n}{r}^2 + ...$$

Generalized Jacobi-Hamilton equation in the weak non-inertial reference frame for the action function takes on the form:

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V + Q, \quad (24)$$

and let us call Q the quantum potential. Here, is the velocity $v = \frac{\partial S}{\partial t} = \frac{\nabla S}{m}$, and the acceleration $a = \dot{v} = \frac{\nabla \dot{S}}{m} = \frac{\nabla^2 S}{m}$, where is the continuity equation $\frac{\partial v}{\partial t} + \nabla v = 0$ for the vector v. Here is $\ddot{v} = \nabla \ddot{S}$.

In the first approximation $Q \approx \alpha_2 \frac{\nabla^2 S}{m} = -\frac{i\hbar}{2m}\nabla^2 S$. (the constant is chosen as $\alpha_2 = \frac{i\hbar}{2}$) is Bohm's quantum potential. Hence, we get the Schrëdinger equation in the first approximation for the function $\psi = Ae^{\frac{i}{\hbar}S}$ from the equation (17) [4]

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V - \frac{i\hbar}{2m}\nabla^2 S$$

Here we complete the Classical Physics with the hidden variables of the real non-inertial reference frames. In this case the weak influence of inertial forces in non-inertial reference frame define the quantum behavior of particles.

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